The performance of the filter can be expressed as the ratio of the weighted ℓ_2 norm of the filter coefficients as shown next in decibels for the case of no modeling errors (i.e., $\Delta \boldsymbol{a} = 0$ and $\Delta \boldsymbol{b} = 0$), and identically distributed independent sensor noise with variance σ_y^2 and identically distributed independent actuator noise with variance σ_u^2.

$$10 \log \left[\frac{E[(z_k - y_k)^2]}{E[(\hat{y}_k - y_k)^2]} \right] = 10 \log \left[\frac{\sigma_y^2(\alpha'' \alpha''^T) + \sigma_u^2(\beta'' \beta''^T)}{\sigma_y^2(\alpha\alpha^T) + \sigma_u^2(\beta\beta^T)} \right] \tag{25}$$

where α'' and β'' are of the same dimensions as α and β and are given by

$$[\alpha'' \ \beta''] = [\alpha \ \beta] \ (I - RN^T[NRN^T]^{-1}N) \tag{26}$$

Conclusions

A finite memory filter is presented for real-time applications, such as smart sensors and/or active sensors, where memory constraints are tight and computationally efficient algorithms are required for implementation on a microchip collocated with the sensor. It is assumed that an ARMA model of the plant dynamics is available. The key concept is the construction of a nonrecursive filter based on weighted averaging of a finite array of past values of the measured inputs and outputs of the plant. Although the filter algorithm has been derived for SISO systems, it can be extended, in principle, to MIMO systems.

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Application of Order-*n* Formulation to Panel Deployment Problem of a Spacecraft

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Introduction

THIS Note presents the application of an order-*n* formulation and intermittent analysis to calculate the deployment of folded multibody systems in space. When a spacecraft is considered as a serial rigid-body system without closed-loop topology, a new and efficient algorithm, an order-*n* formulation,[1] can be applied for the dynamic analysis. This formulation requires only an order of *n* arithmetic operations, where *n* is the number of degrees of freedom of the system. This Note adapts Rosenthal's algorithm[2] for the following situations: 1) a system is free in space, 2) a system has a tree topology, and 3) intermittent motion occurs. In spacecraft dynamics, intermittent motion plays an important role in deployment, docking, mass capture, and mass release. This behavior is formulated using the impulse-momentum equations,[3,4] which are solved recursively by using the order-*n* formulation. A numerical example demonstrates the validity of the present method. A center arm and two panels of a spacecraft model are connected by revolute hinges and are deployed due to the force of a shrunk spring. When the hinge movement is locked by a ratchet mechanism, intermittent motion occurs.

Model Description

Figure 1 shows the deployment sequence of the model that has four rigid bodies, i.e., a main body, a center arm, and two panels. The main body is considered the base body B_0, and both the center arm and the panels are labeled as B_i ($i = 1, 2, 3$). The main body is free in space. The revolute hinges are labeled as H_i ($i = 1, 2, 3$), and the hinge angle is measured as shown in Fig. 1.

Geometrical configuration of tree topology is described by using the body connection array[5] $L(k)$, which represents the label of the adjoining lower numbered body of body B_k. In this model, the body connection array is defined as

$$L(1) = 0, \qquad L(2) = L(3) = 1 \tag{1}$$

The deployment process is illustrated in Figs. 1a–1c. Initially, two panels are folded and attached to the center arm ($q_2 = q_3 = 90$ deg), and then the center arm is folded onto the main body ($q_1 = 90$ deg) as shown in Fig. 1a. At $t = 0$ s, the center arm begins to rotate

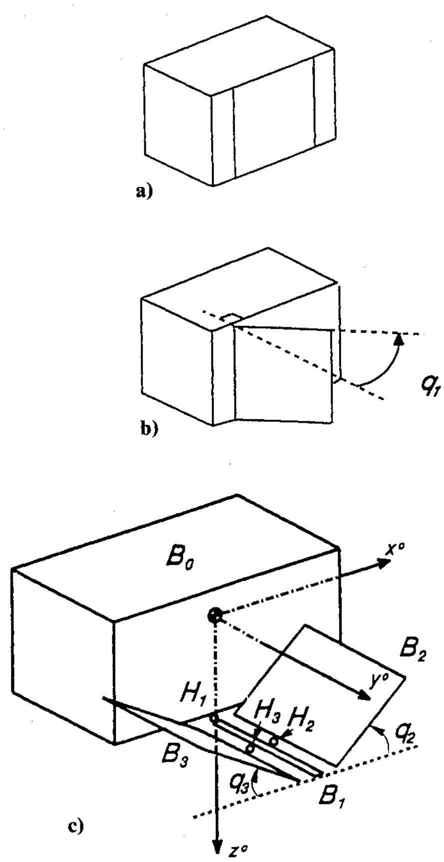

Fig. 1 Spacecraft model and sequence of panel deployment.

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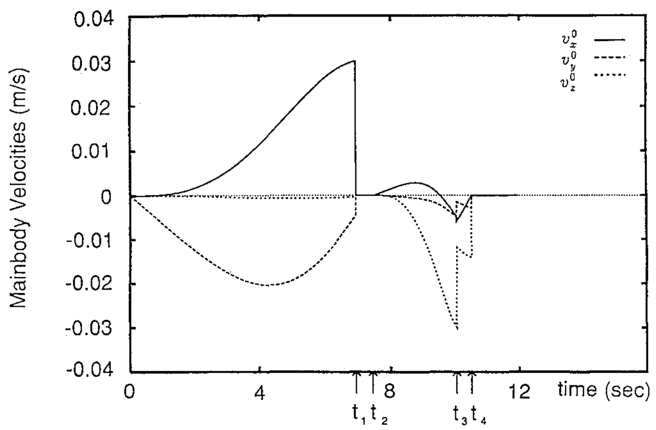
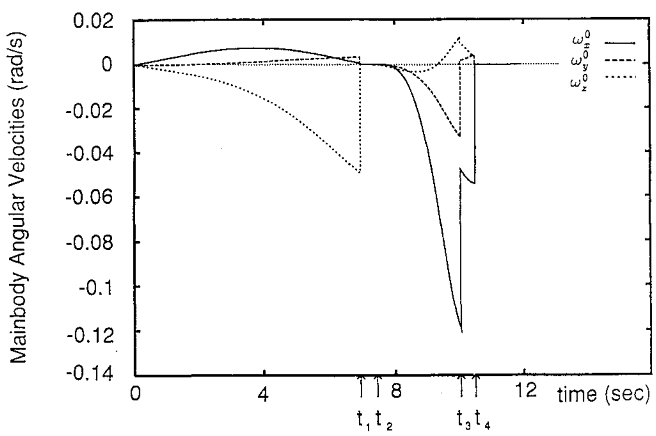
Table 1 Spacecraft model

	Main body	Center arm	Panel 1	Panel 2
Body no., k	0	1	2	3
Mass, kg	480.0	6.0	12.0	12.0
Moment of inertia, kgm^2				
I_{xx}	200.0	0.1	9.0	9.0
I_{yy}	800.0	4.5	13.0	13.0
I_{zz}	680.0	4.5	4.0	4.0
Size, m	4.0×1.0×2.0	3.0×0.0×0.0	2.0×0.0×3.0	2.0×0.0×3.0



a) Velocity of main body's center of gravity



b) Angular velocity of main body

Fig. 2 Time history of main body motion.

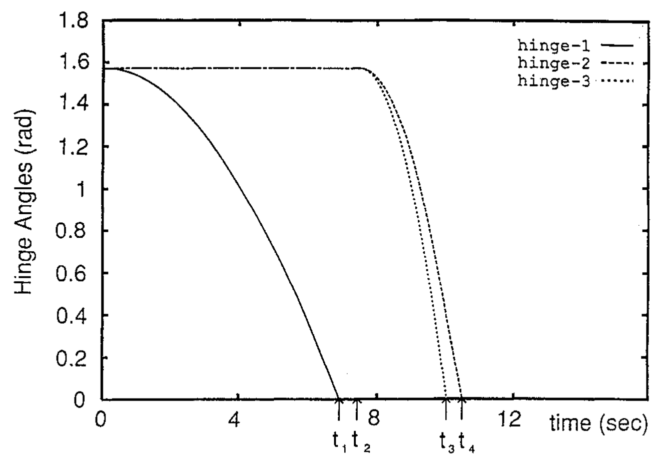


Fig. 3 Time history of hinge angle.

around hinge 1, whereas the two panels are still folded (Fig. 1b). The rotation movement of hinge 1 is stopped when q_1 equals 0 deg. At half a second after the center arm is locked, the two panels begin to rotate around hinge 2 or 3 (Fig. 1c). Their rotations are stopped when q_2 and q_3 are both 0 deg.

Dynamical Equations

Order-n formulations have three recursive passes: a kinematic forward pass to create the kinematic characteristics, a backward pass to obtain the reduced dynamic equations of the base body, and a forward pass to reconstruct the rest of the dynamic equations. Since this Note uses Rosenthal's algorithm[2,6] derived from the formulation of Kane et al.,[7] dynamical equations are expressed by using generalized speeds u_k, i.e., velocities of the base body, its angular velocities, and relative rotation velocity of each hinge.

When some hinges are locked, Rosenthal's algorithm must be modified.[8] At a locked hinge H_k, u_k and $\dot{u}_k$ are both 0, and the hinge torque τ_k is the unknown variable. Thus, in the backward pass, it is not necessary to eliminate $\dot{u}_k$, and τ_k is set to be 0. Furthermore, at a locked hinge, the forward pass must be changed to solve for τ_k instead of $\dot{u}_k$.

When a hinge movement is stopped by a ratchet mechanism, generalized speeds of the system jump discontinuously, and impulsive forces required to stop the hinge rotation are found at locked hinges. The relationship between the jump in generalized speeds and the impulsive forces is formulated by using the impulse-momentum equations. These equations can be derived by integrating the dynamical equations over the infinitely small time interval in which the intermittent motion occurs.[3,4] Note that the impulse-momentum equations have the same form as the dynamical equations: the acceleration of the generalized speed and the force/torque are replaced by the jump in the generalized speed and the impulsive force/torque. Consequently, we are able to solve the impulse-momentum equations by using the order-n formulation.[8]

Numerical Example and Conclusion

Numerical data of the spacecraft model are listed in Table 1. The hinge torque τ_k is written as

$$\tau_k = \varepsilon_k(q_k + q_k^0) + C_k\dot{q}_k \tag{2}$$

where $q_1^0 = q_2^0 = q_3^0 = 0.5$ (rad) are offset angles that produce a torque even when the hinge angle is zero, $\varepsilon_1 = \varepsilon_2 = -3.0$ and $\varepsilon_3 = -4.0$ (N·m/rad) are spring constants, and $C_1 = C_2 = C_3 = -0.2$ (N·m/rad/s) are damping ratios.

The dynamic motion is computed by using the Runge-Kutta-Gill method in which the order-n formulation is employed to calculate the time derivative of generalized speeds. To predict the intermittent motion, we monitor each hinge angle and time in the time integration procedure during the deployment process. When intermittent motion is detected, the time integration is interrupted to solve the impulse-momentum equations. With the use of state variables computed just after the intermittent motion, the dynamical equations are integrated again.

Figure 2a illustrates the time histories of the main body velocities, and Fig. 2b shows the angular velocities. Figure 3 presents the hinge angles. At $t = t_1$, the movement of the center arm is stopped; thus, intermittent motion occurs. At $t = t_2$, hinges 2 and 3 begin to rotate simultaneously. At $t = t_3$, the angle of hinge 3 reaches 0 deg; thus, intermittent motion occurs again. Since hinge 2 continues to rotate, movements of the main body continue after the intermittent motion. At $t = t_4$, hinge 2 is locked, and the deployment is completed. Note that at $t = t_1$ and $t = t_4$ the movement of the system has stopped. This indicates that the conservation law of momentum is satisfied.

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H_2 Approach for Optimally Tuning Passive Vibration Absorbers to Flexible Structures

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Introduction

PASSIVE damping by vibration absorbers has been proven both reliable and successful in many areas of vibration control. Research into the optimal tuning of vibration absorbers has received increased attention lately; however, these tuning methods have only been practical on low-order systems (one or two modes).[1-4]

The design of a vibration absorber system is an optimization problem. In the general case, it is desired to couple multiple vibration absorbers to a structure with multiple vibrational modes. If the total mass of all of the actuators is fixed to some small value compared to the mass of the structure, then the design parameters are the set of absorber stiffnesses and damping coefficients. The problem then becomes the optimization of these parameters with respect to some design criteria. Den Hartog[5] attempted to minimize the maximum value of the transfer function. Juang[2] attempted to minimize a quadratic cost. Both of these approaches give similar tuning values. However, they are only guaranteed to minimize the design criteria for the case of a single vibration absorber tuned to a single vibrational mode. Minimizing the design criteria for the multiple mode and absorber case results is a nonlinear optimization problem. Miller and Crawley[1] experimentally investigated passive absorbers. They validated some general guidelines for tuning vibration absorbers to multiple modes.

There is no straightforward design approach for tuning multiple vibration absorbers to a flexible structure. Also, in some circumstances it may be desirable to design vibration absorbers in the time domain. In this Note it is shown that by using a standard quadratic performance index, output feedback regulator theory provides an approach for tuning vibration absorbers to flexible structures. The technique is computationally attractive, and since it is based on linear quadratic regulator (LQR) design techniques, provides a time domain design method for tuning vibration absorber systems.

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Passive Absorber Tuning by Output Feedback

The following feedback formulation of the passive absorber problem was developed by Posbergh et al.[3]:

$$\dot{x}(t) = Ax(t) + Bf(t) \tag{1}$$

where A and B are the augmented matrices

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_s}{m_s} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \dfrac{1}{m_s} \\ 0 \\ -\dfrac{1}{m_a} \end{bmatrix}$$

where m_s is the mass of the structure, m_a the mass of the actuator, k_s the stiffness of the structure, and $x(t) = [x_s(t)\ \dot{x}_s(t)\ x_a(t)\ \dot{x}_a(t)]^T$. The structure-actuator system is coupled by the output feedback,

$$f(t) = k_a[x_s(t) - x_a(t)] + c_a[\dot{x}_s(t) - \dot{x}_a(t)] \tag{2}$$

which can be put in the matrix form,

$$f(t) = KCx(t) \tag{3}$$

where k_a and c_a are the stiffness and damping of the actuator, and K is a matrix of these values. The matrix C generates the relative position and velocity measurements. Notice that although the feedback formulation was developed based on the single-degree-of-freedom (SDOF) case, it is equally valid for the multiple degree of freedom case.

In this formulation of the passive absorber problem it is interesting to note that tuning the vibration absorber amounts to finding the gains that force the system to meet some design criteria, it is a feedback control problem. A variety of design approaches can be used to solve the problem. The following approach is based on LQR theory.

Given the output feedback problem represented by Eqs. (1) and (3), the passive absorber tuning problem is to find the gain matrix K that minimizes the performance index

$$J = \int_0^\infty x^T(t)Qx(t)\,\mathrm{d}t \tag{4}$$

Because full state feedback is not available, there are natural bounds restricting where the poles can be assigned, and consequently R is set to zero. Also, because passive vibration absorbers are typically performance limited, it is assumed that the desire is to design an absorber that will operate at maximum performance.

Levine and Athans[6] and Kosut[7] investigated the general output feedback problem where $R > 0$. In the specific case where R is zero, it can be shown that the H_2 cost simplifies to

$$J = \mathrm{tr}(P) \tag{5}$$

where P satisfies

$$(A - BCK)^T P + P(A - BKC) + Q = 0 \tag{6}$$

and the necessary conditions for this cost function are

$$B^T V P C^T = 0 \tag{7}$$

where V satisfies

$$(A - BKC)^T V + V(A - BKC) + Q = 0 \tag{8}$$

and P satisfies

$$P(A - BKC)^T + (A - BKC)P + I = 0 \tag{9}$$

Direct use of the necessary conditions to minimize the cost